Dynamical instability vs. thermodynamical stability

Jakub Jankowski

with R. A. Janik, H. Soltanpanahi

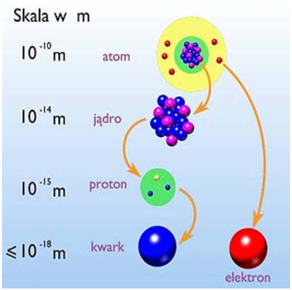
arXiv:1603.05950
arXiv:1512.06871

Institute of Physics
Jagiellonian University

$$\Lambda_{QCD} \approx 200 \text{ MeV} \longleftrightarrow \sim 1 \text{ fm} \approx 10^{-15} \text{ m}$$

QCD		QED
quark	$\longleftrightarrow$	electron
gluon	$\longleftrightarrow$	photon
g_{YM}	$\longleftrightarrow$	e

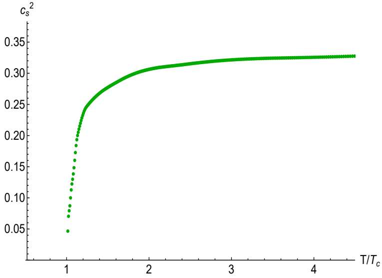
difference: gluons self-interact



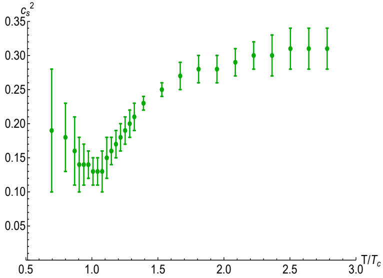
- At low energies quarks and gluons are strongly coupled
- Need for new theoretical techniques to tackle the physics

Phase structure at strong coupling

- Systems at strong coupling exhibit various phase structures
- Pure gluon system $\longrightarrow$ 1^{st} order phase transition (left)
- Gluons + quarks $\longrightarrow$ smooth crossover (right)



G. Boyd *et.al.* Nucl. Phys. B **469**, 419 (1996)



S. Borsanyi *et al.* JHEP **1009**, 073 (2010)

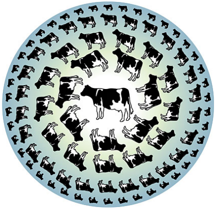
Research directions

- Model different phase structures within strongly coupled models
- Compute the spectrum of linearized perturbations
- Compute transport coefficients and non-hydrodynamic modes
- Check linear stability

Question:
Does dynamical instability has to be accompanied by a thermodynamical instability?

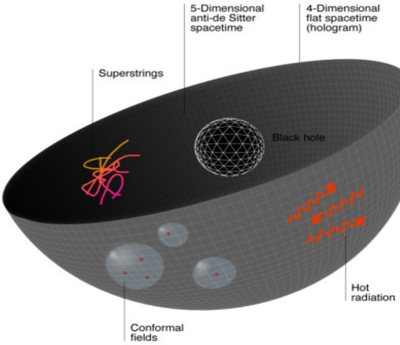
Method:
Use string theory based methods to formulate models at strong coupling!

- Holographic principle
  Quantum gravity in d dimensions must have a number of DOF which scales like that of QFT in $d-1$ dimensions
  't Hooft and Susskind '93



- String Theory realization: *AdS/CFT correspondence*
  Theory is *conformal* and *supersymmetric* Maldacena '97
- Extensions to *non-supersymmetric* and *non-conformal* field theories are possible
- Applications: elementary particle physics and condensed matter physics

Black holes and equilibrium states



Equilibrium state in field theory $\longleftrightarrow$ black hole in dual spacetime

Field theory temperature $\longleftrightarrow$ Hawking temperature

Phase transitions in holography

- To model some non-trivial physics of the boundary theory couple scalar field to gravity theory
- Phase structure is determined by the bulk scalar field interactions quantified by a potential $V(\phi)$
- It is possible to tune parameters to mimic
  $\rightarrow$ crossover e.g. QCD
  $\rightarrow$ 1^{st} order phase transition e.g. pure gluon systems
  $\rightarrow$ 2^{nd} order phase transition

U. Gursoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Linear response and Quasinormal modes

- Perturb the system $\mathcal{L} = \mathcal{L}_0 + h_{ij}\delta^3(x)\delta(t)T^{ij}(x)$ the response is the *retarded Green's* function

$$G_R(\omega, k) \propto i \int dtd^3x\ \theta(t)e^{ikx-i\omega t}\langle[T_{ij}(x,t), T_{kl}(0)]\rangle$$

- *Quasinormal modes*, i.e., solutions of linearized fluctuation equations correspond to poles of holographic retarded Green's functions. In general

$$\omega_n(k) = \Omega_n(k) - i\Gamma_n(k)$$

where $n = 1, 2, 3, \ldots$ $\Omega_n(k)$—oscillation frequency, $\Gamma_n(k)$—damping rate. Stable modes have $\Gamma_n(k) > 0$.

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

Linear response and Quasinormal modes

- Hydrodynamic mode is defined by

$$\lim_{k\to 0} \omega_{\mathrm{H}}(k) = 0$$

- The sound mode

$$\omega(k) = \pm\, c_s\, k - \frac{i}{2\,T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right) k^2 + O(k^3)$$

$\eta-$shear viscosity, $\zeta-$bulk viscosity, $s-$entropy density, c_s-speed of sound, $T-$temperature

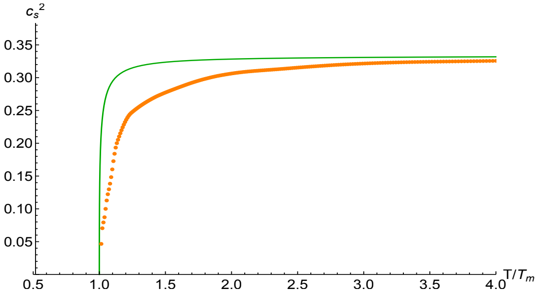
- In holographic models also *non-hydrodynamic* modes are present

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

The first example

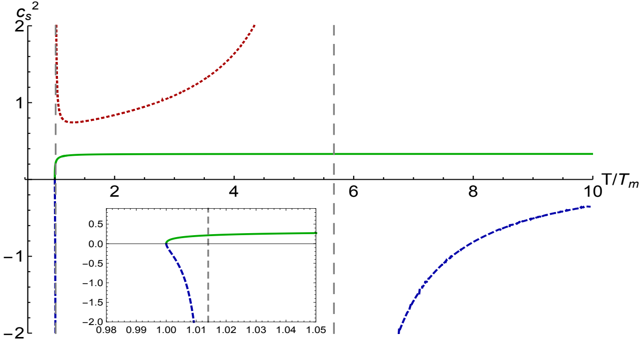
- Holographic model motivated by gluon dynamics
- Transition between black hole and horizon-less geometry

  S. W. Hawking, D. N. Page, Commun. Math. Phys. **87**, 577 (1983)
- Holographic 1^{st} order phase transition



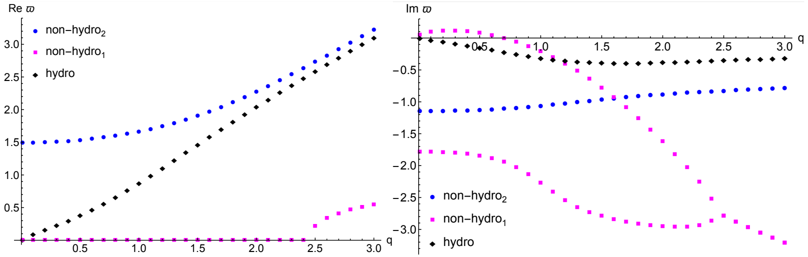
G. Boyd *et.al.* Nucl. Phys. B **469**, 419 (1996)

Full holographic scan



- Below T_m no black hole solution exists
- Various lines represent different black hole phases with different properties

Dynamical instability



- Quasinormal modes at $T = 1.027\,T_m$
- System displays dynamical instability despite thermodynamical stability!

- When $c_s^2 < 0$ we have purely damped hydro-modes

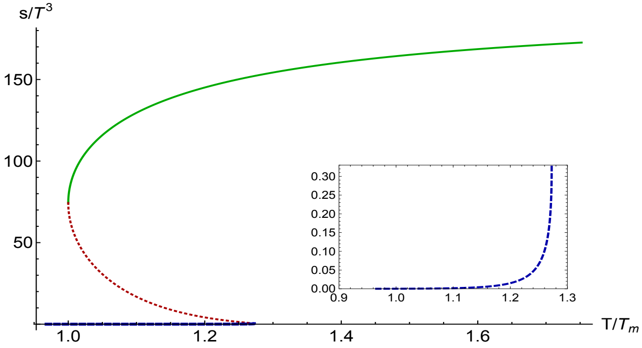
$$\omega \approx \pm i|c_s|\,k - \frac{i}{2\,T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right)k^2$$

so for small enough k we have $\mathrm{Im}\,\omega > 0$

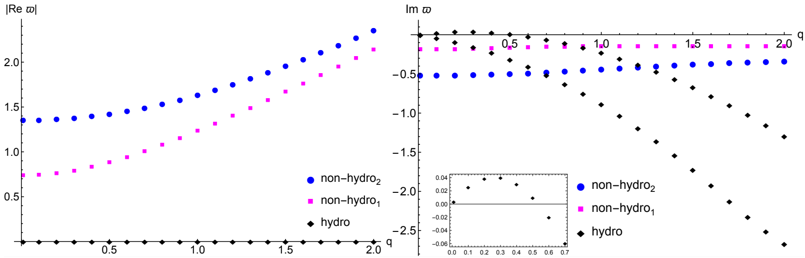
- For a finite range of momenta this mode is present
- This appears for systems with a 1^{st} order phase transition; *spinodal* instability
- This phenomenon occurs e.g. in nuclear matter

P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)

- Transition between two different black hole solutions
- Other example of holographic 1^{st} order phase transition
- As in the previous case there exists minimal temperature T_m
- For the unstable region (red-dashed line) we have $c_s^2 < 0$



Holographic spinodal instability



- Modes for $T \simeq 1.06\, T_m$ where $c_s^2 \simeq -0.1$
- Hydrodynamic mode follows the thermodynamic instability
- Non-hydrodynamic modes have weak momentum dependence

Summary

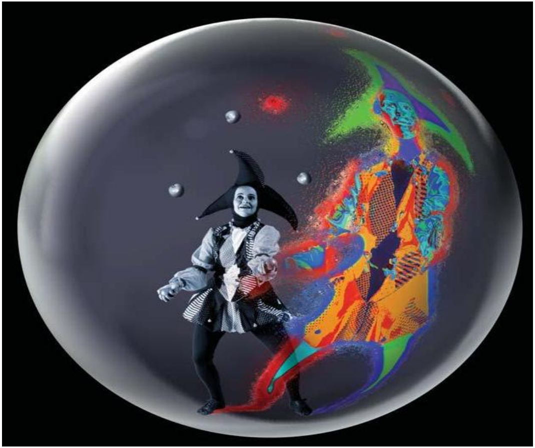
- Thermodynamic instability $\longrightarrow$ dynamical instability
- Converse is not true!

  U. Gursoy, A. Jansen, W. van der Schee, arXiv:1603.07724 [hep-th]
- Non-trivial phase structure limits the applicability of hydrodynamics
- In most cases non-hydro degrees of freedom have very weak dependence on k

Question:

What is field theory interpretation of non-hydrodynamic quasinormal modes?

Thank you!



J. M. Maldacena, *The illusion of gravity,* Scientific American Nov. 2005