Time Crystals

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Outline:

- Time crystals

- Periodically driven systems can reveal crystalline structures in the time domain

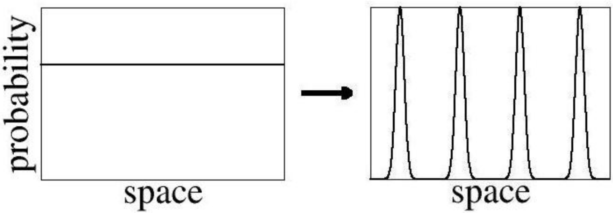
Formation of space crystals

$$[\hat{H}, \hat{T}] = 0$$

$\hat{H}$ – solid state system Hamiltonian
$\hat{T}$ – translation operator of all particles by the same vector

$$\left|\hat{T}\psi\right|^2 = \left|e^{i\alpha}\psi\right|^2 = |\psi|^2$$

t =const.

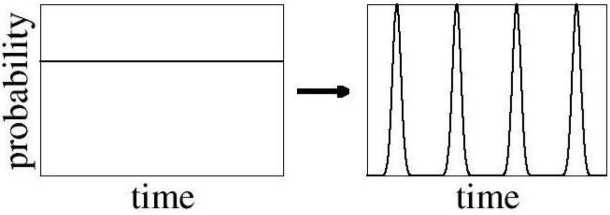


Time crystals

Eigenstates of a time-independent Hamiltonian H are also eigenstates of a time translation operator e^{-iHt}

$$\left|e^{-iHt}\psi\right|^2 = \left|e^{-iEt}\psi\right|^2 = |\psi|^2$$

$\vec{r}$ is fixed



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Modeling time crystals

Single particle systems

Space periodic potential: $H(x+L) = H(x)$

Periodic driving: $H(t+T) = H(t)$

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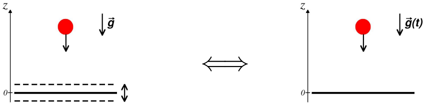
Example: single particle bouncing on an oscillating mirror in 1D:

A. Buchleitner, D. Delande, J. Zakrzewski, Phys. Rep. **368**, 409 (2002).

Floquet Hamiltonian

$$H_F(t) = -\frac{1}{2}\frac{\partial^2}{\partial z^2} + z + \lambda z \cos(2\pi t/T) - i\frac{\partial}{\partial t}$$

$$H_F\ \psi_n(z,t) = E_n\ \psi_n(z,t)$$



E_n – quasi-energy

$\psi_n(z,t)$ – time periodic function

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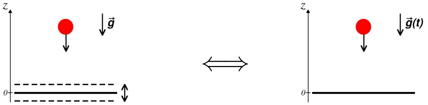
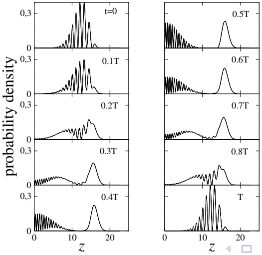


$2:1$ resonance

$\lambda = 0.06,\ T = 5.7$



Modeling time crystals

Single particle systems

In the $s:1$ resonance case:

- There are s Floquet eigenstates with quasi-energies $E_j \approx E_{j'}$.

  These quasi-energies form a band when $s \to \infty$.

- s individual wavepackets, $\phi_j(z,t)$, can be prepared by superposing s Floquet eigenstates.

  For $s \to \infty$, the wavepackets $\phi_j(z,t)$ become analogues of Wannier states but in the time domain.

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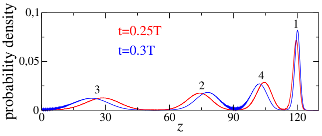
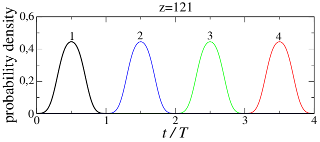
Example for $s = 4$:





Restricting to the Hilbert subspace $\psi = \sum_{j=1}^{s} a_j \phi_j$,

$$E_F = \int_0^{sT} dt \langle \psi | H_F | \psi \rangle \approx -\frac{J}{2} \sum_{j=1}^{s} (a_{j+1}^* a_j + \mathrm{c.c.})$$

$$J = -2 \int_0^{sT} dt \langle \phi_{j+1} | H_F | \phi_j \rangle$$

The lowest and higher quasi-energy bands can be considered.

Anderson localization in the time domain

$$E_F = -\frac{J}{2}\sum_{j=1}^{s}(a_{j+1}^* a_j + \mathrm{c.c.}) + \sum_{j=1}^{s}\varepsilon_j|a_j|^2$$

with $\varepsilon_j = \int_0^{sT} dt \langle\phi_j|H'(t)|\phi_j\rangle$,

where $H'(t)$ is a perturbation that fluctuates in time but $H'(t+sT) = H'(t)$.

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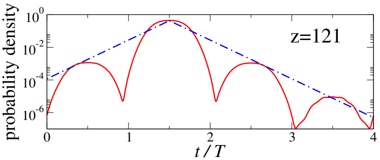

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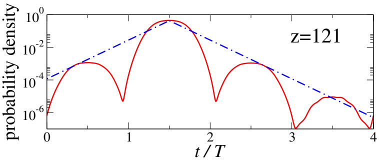
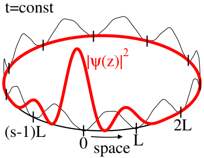
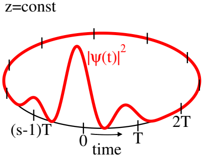


Space Crystal



Time Crystal



Mott insulator-like phase in the time domain

Many-body systems

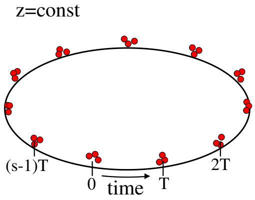
Bosons:

$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}_{j+1}^{\dagger}\hat{a}_j + \mathrm{h.c.}) + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\hat{n}_i\hat{n}_j$$

with $U_{ij} = g_0 \int_0^{sT} dt \int_0^{\infty} dz |\phi_i|^2 \; |\phi_j|^2$, where $|U_{ii}| > |U_{ij}|$ for $i \neq j$.

- For $g_0 \rightarrow 0$, the ground state is a superfluid state with long-time phase coherence.
- For strong repulsion, $U_{ij} \gg NJ/s$, the ground state becomes a Fock state $|N/s, N/s, \ldots, N/s\rangle$ and long-time phase coherence is lost.

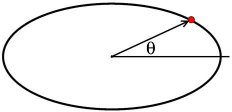
Time Crystal



Anderson localization in the time domain

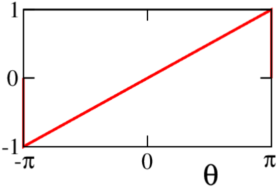
Single particle on a 1D ring

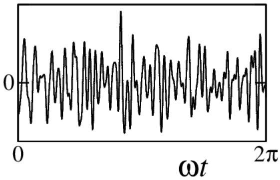
$$H = \frac{p^2}{2} + V\, g(\theta)\, f(t)$$



$$g(\theta) = \frac{\theta}{\pi} = \sum_n g_n e^{in\theta}$$

$$f(t + 2\pi/\omega) = f(t) = \sum_k f_k e^{ik\omega t}$$





Anderson localization in the time domain

Single particle on a 1D ring

In the rotating frame $\tilde{\Theta} = \theta - \omega t$ is a **slow** variable if $\tilde{P} = p - \omega \approx 0$,

$$H_{eff} = \langle H(t) \rangle_t = \frac{\tilde{P}^2}{2} + V \sum_{k=-\infty}^{+\infty} g_k \, f_{-k} \, e^{ik\tilde{\Theta}} + const.$$

Eigenstates $\psi_n(\tilde{\Theta})$ of H_{eff} correspond to **Floquet states**, $(H(t) - i\partial_t)\psi_n = E_n \psi_n$,

where $\psi_n(\theta - \omega t)$ are time-periodic functions.

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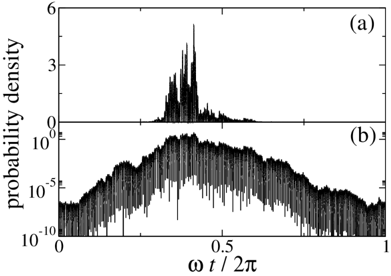
In the lab frame for fixed θ



For example $f(t)$ is so that $|g_k f_{-k}| \propto e^{-k^2/(2k_0^2)}$,
$Arg(f_k)$ is a random variable chosen uniformly in $(0, 2\pi)$,
$k_0 = 10^3$, $V = 4 \cdot 10^3$, $E = 8 \cdot 10^3$.

Localization length in time $l_t = 0.17/\omega$.

Anderson localization in the time domain

Single particle on a 1D ring

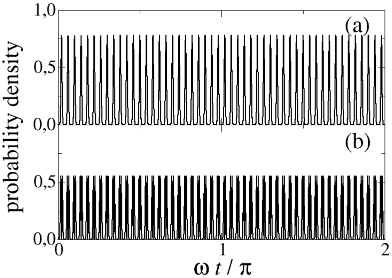
$$H = \frac{p^2}{2} + \lambda \cos(s\theta)\cos(s\omega t).$$

In the rotating frame, $\tilde{\Theta} = \theta - \omega t$,

$$H_{eff} = \frac{\tilde{P}^2}{2} + \frac{\lambda}{2}\cos(s\tilde{\Theta}).$$

In the lab frame for fixed θ

$s = 50$
$\lambda = 2 \cdot 10^4$



Anderson localization in the time domain

Single particle on a 1D ring

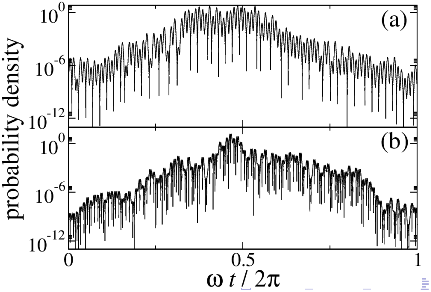
$$H = \frac{p^2}{2} + \lambda \cos(s\theta)\cos(s\omega t) + Vg(\theta)f(t).$$

In the rotating frame, $\tilde{\Theta} = \theta - \omega t$,

$$H_{eff} = \frac{\tilde{P}^2}{2} + \frac{\lambda}{2}\cos(s\tilde{\Theta}) + V\sum_{k=-\infty}^{+\infty} g_k\ f_{-k}\ e^{ik\tilde{\Theta}}.$$

$s = 100$
$\lambda = 2 \cdot 10^4$
$V = 10$ (a)
$V = 300$ (b)
$k_0 = 100$
localization length:
$l_t = 0.16/\omega$

In the lab frame for fixed θ



Summary:

- Time crystals are analogues of space crystals but in the time domain.
- Space periodic potentials are often used to model properties of space crystals. Crystalline structures in the time domain can be modeled by periodically driven systems.
- We show that Anderson localization and Mott insulator-like phase can be observed in the time domain.
- Possible experimental realization:
  - ultra-cold atoms bouncing on an oscillating mirror,
  - electronic motion of Rydberg atoms in microwave fields,
  - ultra-cold gases in ring-shaped traps or superconducting or normal metal devices.

KS, Phys. Rev. A **91**, 033617 (2015).
KS, Sci. Rep. **5**, 10787 (2015).
KS & D. Delande, arXiv:1603.05827

Anderson localization in the time domain

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where $H'(t)$ is a perturbation that fluctuates in time but $H'(t+sT) = H'(t)$.

$$H'(t) = z\sum_{n=1}^{4}\alpha_n \cos\left\{2\pi\left[n\frac{t}{4T} - \sin\left(2\pi\frac{t}{4T}\right)\right]\right\}.$$

ε_j are chosen randomly, e.g. according to a Lorentzian distribution (Lloyd model),

then α_n are chosen so that $\varepsilon_j = \int_0^{sT} dt \langle\phi_j|H'(t)|\phi_j\rangle$.

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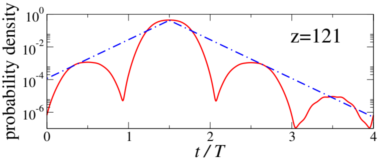

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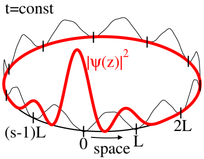

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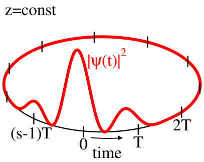

Maximal number of states localized in a *s*-resonant island:

$$n_{max} \approx s\frac{8\sqrt{\lambda}}{\omega^3}.$$

Resonant action

$$I_s = s^3\frac{\pi^2}{3\omega^3}.$$